Q1)a)If A, B and C are events, show that
$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C)$
$-P(B \cap C) + P(A \cap B \cap C)$

b)The traffic police plan to enforce speed limits by using radar traps at 4 different locations within the city limits. The radar traps are located at each of the locations L1,L2,L3 and L4 and are operated $40\%, 30\%, 20\%$ and 30% of the time. A person who is speeding on his way to work has the probability of 0.2, 0.1, 0.5 and 0.2, respectively, of passing through locations L1,L2,L3 and L4. Find the probability that this person will receive a speeding ticket.

Q2)A balanced die is thrown once. If 4 appears, a ball is drawn from urn 1; otherwise a ball is drawn from urn 2. Urn 1 contains 4 red, 3 white and 3 black balls. Urn 2 contains 6 red and 4 white balls.
a) Find the probability that a red ball is drawn.
b)Find the probability that urn 1 was used given that a red ball is drawn.

Q3)3 balls are randomly selected from a bag containing 3 red, 4 white and 5 blue balls. Let X and Y denote, respectively, the number of red and white balls chosen.
a)Find the joint probability mass function $f(x, y)$ of X and Y.
b)Find the marginal probability mass functions of X and Y.
c)Find the probability mass function $f(X|Y = 0)$.
d)Find $P(X \geq 2|Y \leq 1)$.
e)Find $P(Y < 2|X \geq 2)$.

Q4)The joint probability density function of X and Y is given by

$$f(x,y) = \begin{cases} c(y^2 - x^2)e^{-y} & -y \leq x \leq y, 0 < y < \infty \\ 0 & otherwise \end{cases}$$

a) Find c.
b)Find the marginal probability density function of Y.
c))Find $E(Y)$.
d)Find the conditional probability function $f(X|Y)$.

Q1)Two random variable X and Y have the following joint probability function:

$$f(x,y) = \begin{cases} \frac{16y}{x^3} & x > 2, 0 < y < 1 \\ 0 & otherwise \end{cases}$$

a)Find the marginal probability functions of X and Y.
b)Find $E(X)$ and $E(Y)$.
c)Find the covariance σ_{XY}.

Q2)In an NBA playoff series, the team which wins 3 games out of 5 will be the winner. Suppose that team A has the probability of o.6 of winning over team B and teams A and B face each other in the playoff games.
a)What is the probability that team A will win the series in the fourth game?.
b)What is the probability that team A will win the series?.
c)What is the probability that the third match will be the first match that team A wins?.
d)What is the probability that team A will win 2 matches in the series?.

Q3)A random size of 2 is taken from an urn containing two nickels, three dimes and three quarters. The numbers of nickels and dimes in the sample are denoted by X and Y, respectively.
a)Compute the joint probability mass function of X and Y.
b)Find the marginal probability functions of X and Y.
c)Find $E(X)$ and $E(Y)$.
d)Find the covariance σ_{XY}.

Q4)The results of a mathematics exam are assumed to be normally distributed with $\mu = 65$ and $\sigma = 15$.
a)What is the probability that a student scores greater than 75 in the exam?
b)What is the probability that a student scores between 50 and 80? c)In order to get an A, a student must rank in the highest 10%.
What is the minimum score required to get an A?.
d)In order to get an F, a student must rank in the lowest 20%. Below which score if a student scores,he or she will get an F?
e)Ten students are selected at random. What is the probability that at most one of them scored greater than 75.

Q1)a)The probability that a patient recovers from a rare disease is 0.6. If 20 people are known to have contracted the disease, what is the probability that at least 19 recovers?
b)Find the probability that a person flipping a coin gets
(i)the third head on the fifth flip.
(ii)the first head on the fourth flip.
c)A secretary makes 2 errors per page, on average.
(i)What is the probability that on the next page she will make no errors?.
(ii)What is the probability that on the next 2 pages she will make at least two errors?.

Q2)A random variable X is said to have a beta distribution if its probability function is

$$f(x) = \begin{cases} \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)} & 0 < x < 1 \\ 0 & elsewhere \end{cases}$$

where $\alpha, \beta > 0$ and $B(\alpha, \beta) = \displaystyle\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}dx = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$ is called the beta function. Show that the mean and variance of the beta distribution are $\mu = \dfrac{\alpha}{\alpha+\beta}$, $\quad \sigma^2 = \dfrac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$.

Q3)A random variable X has the Poisson distribution
$p(x;\mu) = e^{-\mu}\dfrac{\mu^x}{x!}, \qquad x = 0, 1, 2, ...$
(a)Verify that $M_X(t) = e^{\mu(e^t-1)}$.
(b)Use $M_X(t)$ to find the mean and variance of the Poisson distribution.

Q4)The joint probability function of X_1 and X_2 is given by

$$f(x_1, x_2) = \begin{cases} 24x_1x_2 & 0 < x_1 < 1 - x_2, \quad 0 < x_2 < 1 \\ 0 & otherwise \end{cases}$$

(a)Find the joint probability function of $Y_1 = X_1 + X_2$ and $Y_2 = X_1 - X_2$.
(b)Find the marginal probability function of Y_2.