ÇANKAYA UNIVERSITY

Department of Mathematics and Computer Science

MCS 224 Introduction to Probability and Statistics

1$^{\text{st}}$ Midterm

April 6, 2010

17:40-19:10

Surname : _____

Name : _____

ID # : _____

Department : _____

Section : _____

Instructor : _____

Signature : _____

- The exam consists of 5 questions of equal weight.
- Please read the questions carefully and write your answers under the corresponding questions. Be neat.
- Show all your work. Correct answers without sufficient explanation might <u>not</u> get full credit.
- Calculators are <u>not</u> allowed.

GOOD LUCK!

Please do <u>not</u> write below this line.

Q1	Q2	Q3	Q4	Q5	TOTAL
20	20	20	20	20	100

Question 1. Suppose that we have 3 cards identical in form except that both sides of the first card are colored red, both sides of the second card are colored black, and one side of the third card is colored red and the other side is black. The 3 cards are mixed up in a hat, and 1 card is randomly selected and put down on the ground.

    (a) What is the probability that the upper side of the chosen card is red?   (10 points)

    (b) If the upper side of the chosen card is colored red, what is the probability that the other side is colored black?   (10 points)

Answer 1.

Question 2. Suppose you roll two fair dice and sum the numbers that show. You win 20 TL if a 7 or 11 shows up. You lose 10 TL if a 2, 3, or 12 shows up. For anything else that shows up, you lose 5 TL. Let X be the amount that you win (or loss) in one play of this game.

    (a) Find the probability distribution of X.   (10 points)

    (b) Find the mathematical expectation (mean) of X.   (5 points)

    (b) Find the variance of X.   (5 points)

Answer 2.

Question 3. The probability density function of X, the lifetime of a certain type of electronic device (measured in hours), is given by

$$f(x) = \begin{cases} \dfrac{\alpha}{x^2} & x > 10, \\[2mm] 0 & x \leq 10. \end{cases}$$

(a) Find α. (6 points)

(b) Find $P(X > 20)$. (6 points)

(c) Find the cumulative distribution function of X. (8 points)

Answer 3.

Question 4. The joint probability density function of X and Y is given by

$$f(x,y) = \begin{cases} \frac{6}{7}\left(x^2 + \frac{xy}{2}\right) & 0 < x < 1,\ 0 < y < 2, \\ \\ 0 & \text{elsewhere.} \end{cases}$$

(a) Find $P(X > Y)$.   (5 points)
(b) Find $P(Y > \frac{1}{2}|X < \frac{1}{2})$.   (5 points)
(c) Compute the marginal density function of X.   (5 points)
(d) Find $E(X)$.   (5 points)

Answer 4.

Question 5.

(a) Let X be a continuous random variable and a and b are constants. Prove that
$$\sigma^2_{aX+b} = a^2\sigma^2_X.$$

(10 points)

(b) The length of time, in minutes, for an airplane to obtain clearance for take off at a certain airport is a random variable $g(X) = 3X - 2$, where X has the density function

$$f(x) = \begin{cases} \dfrac{1}{4}e^{-\frac{x}{4}} & x > 0 \\ \\ 0 & \text{elsewhere.} \end{cases}$$

Find the mean and variance of the random variable $g(X)$.

(10 points)

Answer 5.

ÇANKAYA UNIVERSITY

Department of Mathematics and Computer Science

MCS 224 Introduction to Probability and Statistics

2nd Midterm

May 11, 2010

09:00-10:30

Surname : _____

Name : _____

ID # : _____

Department : _____

Section : _____

Instructor : _____

Signature : _____

- The exam consists of 6 questions.
- Please read the questions carefully and write your answers under the corresponding questions. Be neat.
- Show all your work. Correct answers without sufficient explanation might <u>not</u> get full credit.
- Calculators are <u>not</u> allowed.

GOOD LUCK!

Please do <u>not</u> write below this line.

Q1	Q2	Q3	Q4	Q5	Q6	TOTAL
20	20	15	15	15	15	100

Question 1. A simplified programming language uses "words", each consisting of a sequence of 10 digits, either 0 or 1 (for example, 0110101001 is a typical "word"). In transmission, the probability of a digit reversal (0 read as a 1, or vice versa) is 0.01. Digits are read independently.

    (a) Find the probability that a given word is transmitted correctly.   (10 points)

    (b) Find the probability that there will be at least 9 digit reversals.   (10 points)

Answer 1.

Question 2. A foreign student club lists as its members 2 Canadians, 3 Japanese, 5 Italians, and 2 Germans. If a committee of 4 is selected at random, find the probability that

   (a) all nationalities are represented,   (10 points)

   (b) all nationalities except the Italians are represented.   (10 points)

Answer 2.

Question 3. On average a certain intersection results in 3 traffic accidents per month. What is the probability that for any given month at this intersection

    (a) exactly 5 accidents will occur?   (7.5 points)

    (b) less than 3 accidents occur?   (7.5 points)

Answer 3.

Question 4. The probability that a student pilot passes the written test for a private pilot's license is 0.7. Find the probability that the student will pass the test

    (a) on the third try,   (7.5 points)

    (b) before the fourth try.   (7.5 points)

Answer 4.

Question 5. The time it takes a symphony orchestra to play Beethoven's Ninth Symphony has a normal distribution with mean of 64.3 minutes and a standard deviation of 1.15 minutes. The next time it is played, what is the probability that it will take

    (a) between 62.5 and 67.7 minutes?   (5 points)

    (b) more than 68 minutes?   (5 points)

    (c) less than 1 hour?   (5 points)

Answer 5.

Question 6. A university entrance examination is a multiple choice exam consisting 100 questions. Each question has 5 possible answers, only one of them is correct. If a student who takes this exam answers questions by guessing what is the probability that she answers correctly at least 30 questions? (15 points)

Answer 6.

ÇANKAYA UNIVERSITY
Department of Mathematics and Computer Science

MCS 224 Introduction to Probability and Statistics
Final

June 4, 2010

09:00-10:50

Surname   : _____

Name   : _____

ID #   : _____

Department   : _____

Section   : _____

Instructor   : _____

Signature   : _____

- The exam consists of 5 questions of equal weight.
- Please read the questions carefully and write your answers under the corresponding questions. Be neat.
- Show all your work. Correct answers without sufficient explanation might <u>not</u> get full credit.
- Calculators are <u>not</u> allowed.

GOOD LUCK!

Please do <u>not</u> write below this line.

Q1	Q2	Q3	Q4	Q5	TOTAL
20	20	20	20	20	100

Question 1. From the six marbles numbered as shown,

①　①　①　①　②　②

two marbles will be drawn without replacement. Let X denote the sum of the numbers on the selected marbles.

    (a) List the possible values of X and determine the probability distribution.   (10 points)

    (b) Find the expected value of X.   (10 points)

Answer 1.

Question 2. The random variables X and Y have joint density function

$$f(x, y) = \begin{cases} 12xy(1 - x) & 0 < x < 1, \ 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}.$$

(a) Are X and Y independent? (4 points)

(b) Find μ_X. (4 points)

(c) Find μ_Y. (4 points)

(d) Find σ_X^2. (4 points)

(e) Find σ_{XY}^2. (4 points)

Answer 2.

Question 3. Remove all twelve face-cards (J, Q, K) from a deck[1] of cards leaving forty cards A, 2, 3, 4, 5, 6, 7, 8, 9, 10.

    (a) If we draw one card from this deck (of 40 cards), what is the probability that we draw an ace?   (2 points)

    (b) If we draw five cards from this deck of (40 cards), without replacement, what is the probability that we draw at least one ace?   (6 points)

    (c) If we draw five cards from this deck of (40 cards), with replacement, what is the probability that we draw at least one ace?   (6 points)

    (d) If we draw one hundred cards from this deck of (40 cards), with replacement, what is the probability that we draw at least 7 ace?   (6 points)

Answer 3.

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[1]A standard deck of playing cards consists of 52 cards. There are 4 suits, namely, spade (♠) , club (♣), heart (♡) and diamond (◇). For each suit there are 13 denominations, namely, A (ace), 2, 3, 4, 5, 6, 7, 8, 9, 10, J (Jack), Q (Queen), and K (King). Jacks, Queens and Kings are face-cards.

Question 4. There are 36 students in an elementary statistics class. On the basis of years of experience, the instructor knows that the time needed to grade a randomly chosen final examination paper is a random variable with an expected value of 6 min and a standard deviation of 6 min.

    (a) If grading times are independent and the instructor begins grading at 7:06 P.M. and grades continuously, what is the (approximate) probability that she is through grading before the 11:00 P.M. TV news begins?   (10 points)

    (b) If the sports report begins at 11:18 P.M., what is the probability that she misses part of the report if she waits until grading is done before turning on the TV?   (10 points)

Answer 4.

Question 5. A company that makes margarine is trying to develop a new product that has a higher melting point than all other brands. They test 100 samples of their new product and find that the mean melting point is 35°C. The standard deviation of the melting point is known to be $\sigma = 3$°C.

(a) Compute a 95% confidence interval for the true mean melting point of the new product.
(10 points)

(b) The researchers want to repeat the experiment so that the margin of error on the 95% confidence interval is 0.25. What sample size will they need to achieve this? (10 points)

Answer 5.

ÇANKAYA UNIVERSITY
Department of Mathematics and Computer Science

MCS 224 Introduction to Probability and Statistics
Make up for the 2$^{\text{nd}}$ Midterm
June 15, 2010, 11:00-12:50

QUESTIONS

(1) According to *USA Today* (March 18, 1997) of 4 million workers in the general workforce, 5.8% tested positive for drugs. Of those testing positive, 22.5% were cocaine users and 54.4% marijuana users. What is the probability that of 10 workers testing positive,
  (a) 2 are cocaine users, 5 marijuana users, and 3 users of other drugs? (6 points)
  (b) all are marijuana users? (7 points)
  (c) none are cocaine users? (7 points)

(2) A foreign student club lists as its members 2 Canadians, 3 Japanese, 5 Italians, and 2 Germans. If a committee of 4 is selected at random, find the probability that
  (a) all nationalities are represented, (10 points)
  (b) all nationalities except the Italians are represented. (10 points)

(3) On average a certain intersection results in 3 traffic accidents per month. What is the probability that for any given month at this intersection
  (a) exactly 5 accidents will occur? (7 points)
  (b) less than 3 accidents occur? (8 points)

(4) A basketball player makes repeated shots from the free throw line. Assume his shots are independent Bernoulli trials with $p = .7$ (probability that the ball goes through the basket). What is the probability that it takes him less than 5 shots to make his
  (a) first basket, (5 points)
  (b) second basket. (10 points)

(5) A soft drink machine can be regulated so that it discharges an average of μ ounces per cup. If the ounces of fill are normally distributed with standard deviation equal to 0.3 ounce, give the value of μ so that 8-ounce cups will overflow only 1% of the time. (15 points)

(6) Twelve percent of the population is left-handed. Approximate the probability that there are at least 40 left handers in a school of 400 students. (15 points)

ÇANKAYA UNIVERSITY

Department of Mathematics and Computer Science

MCS 224 Introduction to Probability and Statistics

Make up for the Final

June 15, 2010, 11:00-12:50

QUESTIONS

(1) In a carnival game, the player picks a number between 1 and 6. The game operator then rolls three dice. The outcome of the game is determined as follows:
  - If the player's number comes up once, she wins 1 TL.
  - If the player's number comes up twice, she wins 2 TL.
  - If the player's number comes up three times, she wins k TL.
  - If the player's number does not come up at all, she loses 1 TL.
  (a) What is the player's expected profit as a function k? (15 points)
  (b) The game is said to be *fair* to you when your expected gain is 0 YTL. For what value of k is the game fair? (5 points)

(2) The random variables X and Y have joint density function

$$f(x,y) = \begin{cases} 12xy(1-x) & 0 < x < 1,\ 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}.$$

  (a) Are X and Y independent? (4 points)
  (b) Find μ_X. (4 points)
  (c) Find μ_Y. (4 points)
  (d) Find σ_X^2. (4 points)
  (e) Find σ_{XY}^2. (4 points)

(3) Remove all twelve face-cards (J, Q, K) from a deck* of cards leaving forty cards A, 2, 3, 4, 5, 6, 7, 8, 9, 10.
  (a) If we draw one card from this deck (of 40 cards), what is the probability that we draw an ace? (2 points)
  (b) If we draw five cards from this deck of (40 cards), without replacement, what is the probability that we draw at least one ace? (6 points)
  (c) If we draw five cards from this deck of (40 cards), with replacement, what is the probability that we draw at least one ace? (6 points)
  (d) If we draw one hundred cards from this deck of (40 cards), with replacement, what is the probability that we draw at least 7 ace? (6 points)

(4) The weight of an almond is normally distributed with mean $\mu = 0.05$ ounce and standard deviation $\sigma = 0.015$ ounce. Find the probability that
  (a) a randomly selected almond weighs between 0.048 and 0.053 ounce, (4 points)
  (b) the mean weight of 16 randomly selected almonds is between 0.048 and 0.053 ounce, (8 points)
  (c) a package of 100 almonds weigh between 4.8 and 5.3 ounce. (8 points)

(5) A laboratory tested 83 chicken eggs and found that the mean amount of cholesterol was 238 milligrams with standard deviation 13.8 milligrams.
  (a) Construct a 95 percent confidence interval for the true mean cholesterol content, μ, of all such eggs. (10 points)
  (b) How large a sample is needed if researchers wish to be 95% confident that their sample mean will be within 1 milligram of the true mean? (10 points)

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*A standard deck of playing cards consists of 52 cards. There are 4 suits, namely, spade (♠) , club (♣), heart (♡) and diamond (♢). For each suit there are 13 denominations, namely, A (ace), 2, 3, 4, 5, 6, 7, 8, 9, 10, J (Jack), Q (Queen), and K (King). Jacks, Queens and Kings are face-cards.

Area under the standard normal curve from 0 to the point z
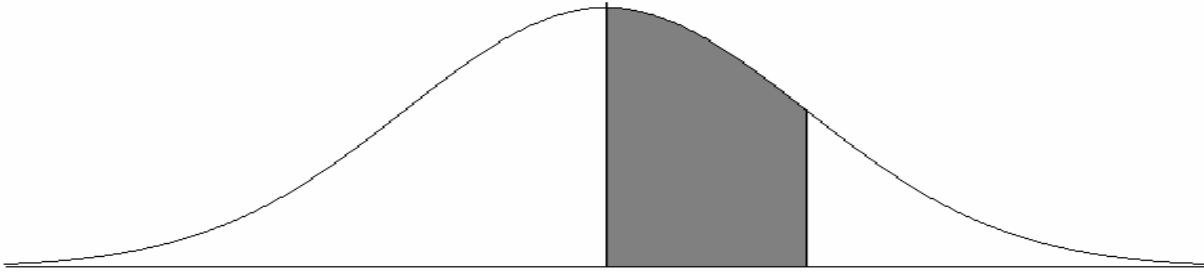
z	0,00	0,01	0,02	0,03	0,04	0,05	0,06	0,07	0,08	0,09
0,0	0,0000	0,0040	0,0080	0,0120	0,0160	0,0199	0,0239	0,0279	0,0319	0,0359
0,1	0,0398	0,0438	0,0478	0,0517	0,0557	0,0596	0,0636	0,0675	0,0714	0,0753
0,2	0,0793	0,0832	0,0871	0,0910	0,0948	0,0987	0,1026	0,1064	0,1103	0,1141
0,3	0,1179	0,1217	0,1255	0,1293	0,1331	0,1368	0,1406	0,1443	0,1480	0,1517
0,4	0,1554	0,1591	0,1628	0,1664	0,1700	0,1736	0,1772	0,1808	0,1844	0,1879
0,5	0,1915	0,1950	0,1985	0,2019	0,2054	0,2088	0,2123	0,2157	0,2190	0,2224
0,6	0,2257	0,2291	0,2324	0,2357	0,2389	0,2422	0,2454	0,2486	0,2517	0,2549
0,7	0,2580	0,2611	0,2642	0,2673	0,2704	0,2734	0,2764	0,2794	0,2823	0,2852
0,8	0,2881	0,2910	0,2939	0,2967	0,2995	0,3023	0,3051	0,3078	0,3106	0,3133
0,9	0,3159	0,3186	0,3212	0,3238	0,3264	0,3289	0,3315	0,3340	0,3365	0,3389
1,0	0,3413	0,3438	0,3461	0,3485	0,3508	0,3531	0,3554	0,3577	0,3599	0,3621
1,1	0,3643	0,3665	0,3686	0,3708	0,3729	0,3749	0,3770	0,3790	0,3810	0,3830
1,2	0,3849	0,3869	0,3888	0,3907	0,3925	0,3944	0,3962	0,3980	0,3997	0,4015
1,3	0,4032	0,4049	0,4066	0,4082	0,4099	0,4115	0,4131	0,4147	0,4162	0,4177
1,4	0,4192	0,4207	0,4222	0,4236	0,4251	0,4265	0,4279	0,4292	0,4306	0,4319
1,5	0,4332	0,4345	0,4357	0,4370	0,4382	0,4394	0,4406	0,4418	0,4429	0,4441
1,6	0,4452	0,4463	0,4474	0,4484	0,4495	0,4505	0,4515	0,4525	0,4535	0,4545
1,7	0,4554	0,4564	0,4573	0,4582	0,4591	0,4599	0,4608	0,4616	0,4625	0,4633
1,8	0,4641	0,4649	0,4656	0,4664	0,4671	0,4678	0,4686	0,4693	0,4699	0,4706
1,9	0,4713	0,4719	0,4726	0,4732	0,4738	0,4744	0,4750	0,4756	0,4761	0,4767
2,0	0,4772	0,4778	0,4783	0,4788	0,4793	0,4798	0,4803	0,4808	0,4812	0,4817
2,1	0,4821	0,4826	0,4830	0,4834	0,4838	0,4842	0,4846	0,4850	0,4854	0,4857
2,2	0,4861	0,4864	0,4868	0,4871	0,4875	0,4878	0,4881	0,4884	0,4887	0,4890
2,3	0,4893	0,4896	0,4898	0,4901	0,4904	0,4906	0,4909	0,4911	0,4913	0,4916
2,4	0,4918	0,4920	0,4922	0,4925	0,4927	0,4929	0,4931	0,4932	0,4934	0,4936
2,5	0,4938	0,4940	0,4941	0,4943	0,4945	0,4946	0,4948	0,4949	0,4951	0,4952
2,6	0,4953	0,4955	0,4956	0,4957	0,4959	0,4960	0,4961	0,4962	0,4963	0,4964
2,7	0,4965	0,4966	0,4967	0,4968	0,4969	0,4970	0,4971	0,4972	0,4973	0,4974
2,8	0,4974	0,4975	0,4976	0,4977	0,4977	0,4978	0,4979	0,4979	0,4980	0,4981
2,9	0,4981	0,4982	0,4982	0,4983	0,4984	0,4984	0,4985	0,4985	0,4986	0,4986
3,0	0,4987	0,4987	0,4987	0,4988	0,4988	0,4989	0,4989	0,4989	0,4990	0,4990
3,1	0,49903	0,49906	0,49910	0,49913	0,49916	0,49918	0,49921	0,49924	0,49926	0,49929
3,2	0,49931	0,49934	0,49936	0,49938	0,49940	0,49942	0,49944	0,49946	0,49948	0,49950
3,3	0,49952	0,49953	0,49955	0,49957	0,49958	0,49960	0,49961	0,49962	0,49964	0,49965
3,4	0,49966	0,49968	0,49969	0,49970	0,49971	0,49972	0,49973	0,49974	0,49975	0,49976
3,5	0,49977	0,49978	0,49978	0,49979	0,49980	0,49981	0,49981	0,49982	0,49983	0,49983
3,6	0,49984	0,49985	0,49985	0,49986	0,49986	0,49987	0,49987	0,49988	0,49988	0,49989
3,7	0,49989	0,49990	0,49990	0,49990	0,49991	0,49991	0,49992	0,49992	0,49992	0,49992
3,8	0,49993	0,49993	0,49993	0,49994	0,49994	0,49994	0,49994	0,49995	0,49995	0,49995
3,9	0,49995	0,49995	0,49996	0,49996	0,49996	0,49996	0,49996	0,49996	0,49997	0,49997
4,0	0,49997	0,49997	0,49997	0,49997	0,49997	0,49997	0,49998	0,49998	0,49998	0,49998