Introduction to Probability and Statistics
FIRST MIDTERM

1) The following data represents the grades of a group of students:
10,17,22,34,44,44,48,49,55,59,60,63,70,81,83,89,90

    a) Find the mean
    b) Find the median
    c) Find the standard deviation
    d) How many standard deviation is the top grade above the mean?

2) Gören, Bengisu, Tuncay, Nurşah, Bahadır and Merve are sitting in a row.
    a) In how many different ways can they sit if boys and girls alternate?
    b) In how many different ways can they sit if Merve and Bahadır are to sit next to each other but Gören and Nurşah are not to sit next to each other?

3) There are three departments in a Faculty: Math, English and Translation. The Math department has 3 times as many students as the Translation, and 2 times as many students as the English department. 40% of all students in the Math department, 80% of English department and 90% of the Translation department are girls. We choose a student from the Faculty at random. Given that the student is a girl, what is the probability that she's a Math student?

4) We pick 3 cards from a deck randomly. We win if they are all the same color (red or black) or if each of the numbers are less than or equal to 3. Find the probability of winning.

5) X is a continuous random variable with probability density function

$$f(x) = \begin{cases} k(1-x)^2 & \text{if} \quad 0 \leqslant x \leqslant 1 \\ 0 & \text{if} \quad x < 0 \text{ or } x > 1 \end{cases}$$

    a) Find k
    b) Find the $P(0.75 < X)$.

Introduction to Probability and Statistics
SECOND MIDTERM

1) Given the joint density function $f(x) = \begin{cases} \dfrac{6 - x - y}{8} & 0 < x < 2,\ 2 < y < 4 \\ 0 & \text{elsewhere} \end{cases}$

    a) find marginal distribution of X
    b) find marginal distribution of Y
    c) Are they independent?

2) Let X have the density function $f(x) = \begin{cases} \dfrac{8}{x^3} & \text{if } 2 < x, \\ 0 & \text{if } x < 2 \end{cases}$

    a) Find μ
    b) Find $E\big[(X - 2)^2\big]$

3) The probability that a student is ill on the day of an exam is $\dfrac{1}{20}$. A student has 8 examinations this month on separate days. What is the probability that the student will be ill
    a) on none of the exams?
    b) on exactly two exams?
    b) on more than two exams?

4) We examine a shipment of 200 laptops for defects by choosing a sample of 20 at random. We reject the shipment if there are 2 or more defective ones in the sample. Assume there are 10 defective ones among the 200. What is the probability that we will
    a) reject the shipment?
    b) accept the shipment?

5) On average there are 7 snowstorms per year in a certain area. What is the probability that there are
    a) Exactly 7
    b) More than 7
    c) Less than 7
    snowstorms in a given year?

6) MCS 224 grades have a normal distribution with (hopefully) $\mu = 55$ and $\sigma = 12$. Emre hoca wants to fail only the bottom 20% of the class. What should the passing grade be?

Introduction to Probability and Statistics
FINAL

1) Given the joint density function $f(x) = \begin{cases} \dfrac{3}{2}(x^2 + y^2) & 0 \leqslant x \leqslant 1,\ 0 \leqslant y \leqslant 1 \\ 0 & \text{elsewhere} \end{cases}$

Compute the probability that $0 \leqslant x \leqslant 1/2,\ 0 \leqslant y \leqslant 1/2$

2) The average number of hurricanes per year is 12. What is the probability that on 3 of the next four years, there are 8 or fewer hurricanes?

3) Suppose that airplane engines operate independently and fail with probability equal to 0.4. A plane makes safe flight if at least half of its engines run. Determine whether a 4-engine plane or 2-engine plane has higher probability of successful flight.

4) The average time for a trip is 30 minutes with a standard deviation of 4 minutes. Assume distribution is normal.
a) Find the probability that a trip takes longer than 40 minutes.
b) The longest 10% of the trips take at least how many minutes?

5) The lifetime in weeks of a certain type of transistor is known to follow a gamma distribution with mean 10 weeks and standard deviation $\sqrt{50}$ weeks. What is the probability that the transistor will last at most 50 weeks?

6) A certain cable is manufactured with mean strength of 78.3 kilograms and standard deviation 5.6 kilograms. Find the variance of sample mean if sample size is
a) 64
b) 196

Name-Surname: 24.03.2011

ID Number:

CLASSWORK 1

1) Let (X, Y) have the joint probability function specified in the table.

   a) Compute $P(X = Y)$ **b)** Compute $P(Y > 2)$ **c)** Compute $P(X \geqslant 1, Y \leqslant 3)$

		Y			
		2	3	4	5
	0	0.04	0.08	0.04	0.08
X	1	0.08	0.08	0.20	0.08
	2	0.08	0.04	0.08	0.12

2) Let (X, Y) have joint probability function given below. Compute $P(X + Y \leqslant 1)$

$$f(x, y) = \tfrac{6}{7}(x + y)^2, \quad 0 \leqslant x \leqslant 1,\ 0 \leqslant y \leqslant 1$$
$$f(x, y) = 0 \qquad \text{elsewhere}$$

Name-Surname: 31.03.2011

ID Number:

CLASSWORK 2

1) Let X has the density function $f(x) = \begin{cases} \dfrac{32}{(x+4)^3} & x > 0 \\ 0 & \text{elsewhere} \end{cases}$

Find the expected value of X.

2) The random variable X represents the number of defective items in a lot of 1000. It has the probability distribution given in the table. Find the variance of X.

x	0	1	2	3	4
$f(x)$	0.02	0.08	0.3	0.45	0.15

Name-Surname: 07.04.2011

ID Number:

CLASSWORK 3

1) If the joint density function of X and Y is given by:

$$f(x) = \begin{cases} \dfrac{2}{7}(x + 2y) & 0 < x < 1, \ 1 < y < 2 \\ 0 & \text{elsewhere} \end{cases}$$

find the expected value of $g(X, Y) = \dfrac{X^2}{Y^3}$

Name-Surname: 14.04.2011
ID Number:

CLASSWORK 4

We consider purchasing a welding machine. An efficient machine is successful at 99% of the jobs. An inefficient machine is successful at 95% of the jobs. We perform 100 welds with each machine and purchase the ones that are successful at 97 or more welds.
a) What is the probability that we will reject an efficient machine?
b) What is the probability that we will accept an inefficient machine?

Name-Surname: 21.04.2011

ID Number:

CLASSWORK 5

1) Two chess players A and B will meet in 9 matches. The one to win 5 matches wins the tournament. The probability that A will win in a single match is 0.7. What is the probability that B wins the tournament in 9 matches?

2) On average, there are 6 misprints per a report. What is the probability that a given report
   a) has no misprints?
   b) has more than 5 misprints?

Name-Surname: 05.05.2011

ID Number:

CLASSWORK 6

MCS 224 grades have a normal distribution with $\mu = 53$ and $\sigma = 15$. Emre hoca wants to give CC to middle 30% of the class. What should the limits for CC be?