ÇANKAYA UNIVERSITY

Department of Mathematics and Computer Science

MCS 224 - Introduction to Probability and Statistics

FIRST MIDTERM EXAMINATION
29.03.2012

STUDENT NUMBER:

NAME-SURNAME:

SIGNATURE:

INSTRUCTOR: Seçil Gergün

DURATION: 110 minutes

Question	Grade	Out of
1		20
2		20
3		20
4		20
5		20
Bonus		10
Total		110

IMPORTANT NOTES:

1) Please make sure that you have written your student number and name above.

2) Check that the exam paper contains 6 problems.

3) Show all your work. No points will be given to correct answers without reasonable work.

Question 1.

(a) A gambler has in his pocket a fair coin and a two-headed coin. He selects one of the coins at random, and when he flips it, it shows heads. What is the probability that it is the fair coin? (8 points)

(b) Suppose that he flips the same coin a second time and again it shows heads. Now what is the probability that it is the fair coin? (8 points)

(c) Suppose that he flips the same coin a third time and it shows tails. Now what is the probability that it is the fair coin? (4 points)

Answer 1.

Question 2.

(a) What is the total number of possible outcomes of the roll of two dice? and three dice? (4 points)

(b) Which is more likely: rolling a total of 8 when two dice are rolled or rolling a total of 8 when three dice are rolled? (8 points)

(c) Which is more likely: rolling a total of 9 when two dice are rolled or rolling a total of 9 when three dice are rolled? (8 points)

Answer 2.

Question 3.

(a) Let X be random variable having the density function

$$f(x) = \begin{cases} \dfrac{4}{\pi(1+x^2)}, & 0 < x < 1, \\ 0 & \text{elsewhere.} \end{cases}$$

Find the expected value of X. (10 points)

(b) An urn contains 10 balls of which 1 is black. Let X be the number of draws, without replacement, necessary to observe the black ball. Find the probability function and the mean of X. (10 points)

Answer 3.

Question 4. The joint probability density function of X and Y is given by

$$f(x,y) = \begin{cases} \alpha(1-|x-y|) & 0 < x < 1,\ 0 < y < 1, \\ \\ 0 & \text{elsewhere.} \end{cases}$$

(a) Find α. (8 points)

(b) Find the marginal disrtibution of X. (8 points)

(c) Find $P(X > Y)$. (4 points)

Answer 4.

Question 5. Let X denote the number of times a certain numerical control machine will malfunction: 1, 2, or 3 times on any given day. Let Y denote the number of times a technician is called on an emergency call. Their joint probability distribution is given as

		x		
	$f(x,y)$	1	2	3
	1	0.05	0.05	0.1
y	2	0.05	0.1	0.35
	3	0	0.2	0.1

(a) Evaluate the marginal distribution of X. (5 points)

(b) Evaluate the marginal distribution of Y. (5 points)

(c) Find $P(Y = 3 \big| X = 2)$. (5 points)

(d) Are X and Y statistically independent? (5 points)

Answer 5.

Bonus. Romeo and Juliet have a date at a given time, and each will arrive at the meeting place with a delay between 0 and 1 hour, with all pairs of delays being equally likely. The first to arrive will wait for 15 minutes and will leave if the other has not yet arrived. What is the probability that they will meet? (10 points)

Answer.

ÇANKAYA UNIVERSITY

Department of Mathematics and Computer Science

MCS 224 - Introduction to Probability and Statistics

SECOND MIDTERM EXAMINATION
26.04.2012

STUDENT NUMBER:

NAME-SURNAME:

SIGNATURE:

INSTRUCTOR: Seçil Gergün

DURATION: 110 minutes

Question	Grade	Out of
1		20
2		20
3		20
4		20
5		20
Bonus		20
Total		120

IMPORTANT NOTES:

1) Please make sure that you have written your student number and name above.

2) Check that the exam paper contains 6 problems.

3) Show all your work. No points will be given to correct answers without reasonable work.

Question 1.

(a) A family has 6 children. Find the probability* that there are

(i) 3 boys and 3 girls, (6 points)

(ii) fewer boys than girls. (8 points)

(b) The painted light bulbs produced by a company are 50% red, 30% green and 20% blue. In a sample of 5 bulbs, find the probability that 2 are red, 1 is green and 2 are blue. (6 points)

Answer 1.

*Assume that a boy or a girl is equally likely to occur and the sex of any child is independent of any brothers or sisters.

Question 2. Suppose there is an average of 2 suicides per year per 50 000 population. In a city of 100 000 find the probability that in a given year there are

(a) 0, (5 points)

(b) 1, (5 points)

(c) 2, (5 points)

(d) 2 or more suicides. (5 points)

Answer 2.

Question 3.

(a) Let repeated independent trials result in a success with probability p and a failure with probability $q = 1 - p$ and let X be the number of the trial on which the kth success occurs. Find the expected value of X. (10 points)

(b) Let X be a continuous random variable and a and b are constants. Prove that

$$\sigma^2_{aX+b} = a^2\sigma^2_X.$$

(10 points)

Answer 3.

Question 4. An oil company conducts a geological study that indicates that an exploratory oil well should have a 20% chance of striking oil.

(a) What is the probability that the first strike comes on the third well drilled? (8 points)

(b) What is the probability that the third strike comes on the seventh well drilled? (8 points)

(c) What is the expected number of wells that must be drilled if the oil company wants to set up three producing wells? (4 points)

Answer 4.

Question 5. Suppose that the travel time from your home to the New Campus is normally distributed with mean 40 minutes and standard deviation 7 minutes.

(a) Find the probability that it will take you between 34 and 37 minutes to come to the New Campus from home. (5 points)

(b) Find the probability that it will take you more than 45 minutes to come to the New Campus from home. (5 points)

(c) Find the probability that it will take you less than 40 minutes to come to the New Campus from home. (5 points)

(b) If you want to be 95 percent certain that you will not be late for an office appointment at 2 P.M., what is the latest time that you should leave home? (5 points)

Answer 5.

Bonus. In a certain card game, each card has a point value.

- Numbered cards in the range 2 to 9 are worth five points each.
- The card numbered 10 and the face cards (jack, queen, king) are worth ten points each.
- Aces are worth fifteen points each.

(a) Suppose that you thoroughly shuffle a 52-card deck. What is the expected total point value of the two cards on the top of the deck after the shuffle? (10 points)

(b) Suppose that you throw out all the cards except diamonds and shuffle the remaining 13-card. Now what is the expected total point value of the top two cards? (10 points)

Answer.

ÇANKAYA UNIVERSITY

Department of Mathematics and Computer Science

MCS 224 - Introduction to Probability and Statistics

FINAL EXAMINATION
29.05.2012

STUDENT NUMBER:

NAME-SURNAME:

SIGNATURE:

INSTRUCTOR: Seçil Gergün

DURATION: 110 minutes

Question	Grade	Out of
1		20
2		20
3		20
4		20
5		20
Total		100

IMPORTANT NOTES:

1) Please make sure that you have written your student number and name above.

2) Check that the exam paper contains 5 problems.

3) Show all your work. No points will be given to correct answers without reasonable work.

Question 1. Two types of coins are produced at a factory: a fair coin and a biased one that comes up heads 55 percent of the time. We have one of these coins but do not know whether it is a fair coin or a biased one. In order to ascertain which type of coin we have, we shall perform the following statistical test: We shall toss the coin 100 times. If the coin lands on heads 55 or more times, then we shall conclude that it is a biased coin, whereas, if it lands heads less than 55 times, then we shall conclude that it is the fair coin. What is the probability that we shall reach a false conclusion, if the coin is actually fair? (20 points)

Answer 1.

Question 2. The CPU time requirement of a typical program measured in minutes is found to follow a Gamma Distribution with parameters $\alpha = 3$ and $\beta = 2$. What is the probability that CPU demand of a program will exceed 1 minute? (20 points)

Answer 2.

Question 3. Assume that automobile accidents in Turkey over a holiday period, occur according to a Poisson process with rate 10 per hour. Let Y be the time until the first accident from the beginning of the holiday period.

(a) Find the mean and standard deviation of Y. (10 points)

(b) Find the probability that Y exceeds 15 minutes. (10 points)

Answer 3.

Question 4. Assume the average weight of students, faculty and staff of Çankaya University is 68 kilograms, the standard deviation is 12 kilograms, and that the distribution of weights of individuals is approximately normal.

(a) A random sample of 4 persons from campus is taken, and their weights are measured. What is the probability that the average weight of these randomly selected persons exceed 56 kilograms? (10 points)

(b) In R Block of university campus, there is a sign in the elevator that indicates a limit of 9 persons. Furthermore, there is a weight limit of 720 kilograms. What is the probability that a random sample of 9 persons on the elevator will exceeds the weight limit? (10 points)

Answer 4.

Question 5. A large hospital finds that in 100 randomly selected days it had, on average, $\bar{x} = 96.4$ patient admissions per day, with a standard deviation of $s = 12.2$. Construct a 95% confidence interval for the actual daily average number of hospital admissions. (20 points)

Answer 5.

ÇANKAYA UNIVERSITY

Department of Mathematics and Computer Science

MCS 224 - Introduction to Probability and Statistics

Make-Up EXAMINATION
11.06.2012

STUDENT NUMBER:

NAME-SURNAME:

SIGNATURE:

INSTRUCTOR: Seçil Gergün

DURATION: 110 minutes

Question	Grade	Out of
1		20
2		20
3		20
4		20
5		20
Total		100

IMPORTANT NOTES:

1) Please make sure that you have written your student number and name above.

2) Check that the exam paper contains 5 problems.

3) Show all your work. No points will be given to correct answers without reasonable work.

Question 1.

(a) An urn contains 3 red marbles and 7 white marbles. A marble is drawn from the urn and a marble of the other color is then put into the urn. A second marble is drawn from the urn. If both marbles were the same color, what is the probability that they were both white? (10 points)

(b) A family has 6 children. Assume that a boy or a girl is equally likely to occur and the sex of any child is independent of any brothers or sisters. Find the probability that there are fewer boys than girls. (10 points)

Answer 1.

Question 2. The joint probability density function of X and Y is given by

$$f(x,y) = \begin{cases} \alpha(1-|x-y|) & 0 < x < 1,\ 0 < y < 1, \\ \\ 0 & \text{elsewhere.} \end{cases}$$

(a) Find α. (8 points)

(b) Find the marginal disrtibution of X. (8 points)

(c) Find $P(X > Y)$. (4 points)

Answer 2.

Question 3.

(a) Suppose there is an average of 2 suicides per year per 50 000 population. In a city of 100 000 find the probability that in a given year there are 2 or more suicides. (10 points)

(b) An urn contains 10 balls of which 1 is black. Let X be the number of draws, without replacement, necessary to observe the black ball. Find the probability function and the mean of X. (10 points)

Answer 3.

Question 4.

(a) An oil company conducts a geological study that indicates that an exploratory oil well should have a 20% chance of striking oil. What is the probability that the third strike comes on the seventh well drilled? (10 points)

(b) Suppose that an industrial organization has three vacancies of management position which are to be filled. There are 10 applicants, six of them white and four of them blacks. Let X be the number of blacks selected. Find the probability distribution and the expected value of X. (10 points)

Answer 4.

Question 5. Suppose that the travel time from your home to the New Campus is normally distributed with mean 40 minutes and standard deviation 7 minutes.

(a) Find the probability that it will take you between 34 and 37 minutes to come to the New Campus from home. (5 points)

(b) Find the probability that it will take you more than 45 minutes to come to the New Campus from home. (5 points)

(c) Find the probability that it will take you less than 40 minutes to come to the New Campus from home. (5 points)

(b) If you want to be 95 percent certain that you will not be late for an office appointment at 2 P.M., what is the latest time that you should leave home? (5 points)

Answer 5.