MCS 224 2011-2012 Spring
Exercise Set II

1. Five balls, numbered 1,2,3,4, and 5, are placed in an urn. A random sample of two balls are selected without replacement, and their numbers noted (order of the numbers is not important). Let X be the largest of the two sampled numbers.
   (a) Find the probability distribution of X.
   (b) Find the expected value of X.
   (c) Find the variance of X.

2. Accident record collected by an automobile give the following information. The probability that an insured driver has an automobile accident is 0.15. If an accident has occurred, the damage to the vehicle amounts to 20% of its market value with a probability of 0.80, to 60% of its market value with a probability of 0.12, and to a total loss with a probability of 0.08. What premium should the company charge on a \$4000 car so that the expected gain by the company is zero?

3. Suppose the probability function of a random variable Y is given by the formula

$$f(y) = \frac{y}{10} \qquad y = 1, 2, 3, 4$$

   Calculate the mean and the standard deviation of random variable Y.

4. A trip insurance policy pays \$1000 to the customer in case of a loss due to theft or damage on a five-day trip. If the probability of occurrence of such a loss is 0.005 and customer pays \$10 to buy such policy, find the expected profit per policy to the company.

5. Suppose that an antique jewelry dealer is interested in purchasing a gold necklace for which the probabilities are 0.22, 0.36, 0.28, and 0.14, respectively, that she will be able to sell it for a profit of \$250, sell it for a profit of \$150, break even, or sell it for a loss of \$150. What is her expected profit?

6. In bidding for a remodeling project, a carpenter determines that he will have a net profit of \$5000 if he gets the contract and a net loss of \$56 if his bid fails. If the probability of his getting the contract is 0.25, calculate his expected return.

7. Let Y be a random variable with the probability distribution given in the accompanying table. Find $E(Y)$, $E(Y^2 - 1)$ and $\text{Var}(Y)$.

y	1	2	3	4
$f(y)$	0.4	0.3	0.2	0.1

8. An insurance company writes a policy to the effect that an amount of money $A =$ \$6000 must be paid if some event E occurs within a year. If the company estimates that E will occur within a year with probability 0.02, what should it charge the customer in order that its expected profit will be 10% of A?

9. The owner of construction company A makes bids on jobs so that if awarded the job, company A will make a \$10,000 profit. The owner of company B makes bids on jobs so that if awarded the job, company B will make a \$15,000 profit Each company describes the probability distribution of the number of jobs the company is awarded per year as shown in the table:

Company A		Company B	
2	0.05	2	0.15
3	0.15	3	0.30
4	0.20	4	0.30
5	0.35	5	0.20
6	0.25	6	0.05

   (a) Find the total expected number of jobs awarded by companies A and B in a year.

   (b) What is the expected profit for each company?

10. The manager of stockroom in a factory has constructed the following probability distribution for the daily demand (the number of times used) for a particular tool:

y	0	1	2
$P(y)$	0.1	0.5	0.4

   If it costs the factory \$10 each time the tool is used, find the mean and the variance of the daily costs for use of the tool.

11. A lottery is going to give away a 3000-dollar car. They sell 10000 tickets at 1 dollar a piece. If you buy 1 ticket, what is your expected gain? What is your expected gain if you buy 100 tickets? Compute the variance of your gain in these two instances.

12. Let X be a number of products of a company demanded weekly. The table below gives the probability function of X.

X	0	1	2	3	4	5
$P(X = x)$	0.05	0.20	0.40	0.24	0.10	0.01

   Determine the following:

   (a) Expected weekly demand

   (b) Variance of weekly demand

   (c) Standard deviation of weekly demand.

13. A fair coin and a fair die are tossed. Let X be the sum of the number of heads and the number of dots that die show. Determine:

   (a) the distribution function of X.

(b) expected value of X.

14. An oil company determines that probability of success of a gas station located along highway between two major cities is 0.55. A successful station earns an annual profit of 40000 dollars; one that is not loses 10000 dollars annually. What is the expected gain to the company if it locates a station along a highway between two major cities?

15. Let X be a random variable with the following probability distribution:

x	$P(X = x)$
0	$\frac{1}{8}$
1	$\frac{1}{4}$
2	$\frac{3}{8}$
3	$\frac{1}{4}$

Find the mean and standard deviation of X.

16. The number of customers per day at a certain sales counter denoted by Y, has been observed for a long period of time and found to have a mean of 20 customers with a standard deviation of 2 customers. The probability distribution of Y is not known. What can be said about the probability that Y will be between 16 and 24 tomorrow?

17. A manufacturer of floor wax has developed two new brands, A and B, which she wishes to subject to a housewife evaluation to determine which of the two is superior. Both waxes, A and B are applied to floor surfaces in each of 10 homes. Assume that there is actually no difference in the quality of the brands.

    (a) What is the probability that at least 8 would state a preference for brand A?

    (b) What is the probability that at least 8 would state a preference for either brand A or brand B?

    (c) What is the expected number housewives who would state a preference for brand A?

18. A simplified programming language uses "words", each consisting of a sequence of 10 digits, either 0 or 1 (for example, 0110101001 is a typical "word"). In transmission, the probability of a digit reversal (0 read as a 1, or vice versa) is 0.01. Digits are read independently.

    (a) Find the probability that a given word is transmitted correctly.

    (b) Find the probability that there will be at least 9 digit reversals.

19. In studying birthdays, it is assumed that one is just as likely to be born on Sunday as on any other day. 10 individuals are to be randomly selected, what is the probability that at least two were born on Sunday? What is the probability that exactly 7 were born on Sunday?

20. In a lottery, you pay \$0.50 to choose a number (integer) between 0 and 9999, inclusive. If that number is drawn, you win \$2500. What is your expected gain (or loss) per play? If you play the lottery 5 times what would be your expected gain?

21. Suppose that a telephone solicitor for a particular charity makes 18 calls an hour. She estimates that there is a 20% chance that a given call will result in a donation. The responses to the calls are assumed to be independent. Let X denote the number of donations obtained per hour.

    (a) Find the probability that no donations will be obtained during a given hour.

    (b) Find the probability that at least one donation will be obtained during a given hour.

    (c) Find $E(X)$, σ^2 and σ.

    (d) If the donations average \$10 each, what amount of money does the solicitor expect to raise per hour on the average?

22. A commuter's drive to work includes 7 stoplights. Assume the probability that a light is red when the commuter reaches it is 0.20, and the lights are far enough apart to operate independently. If X is the number of red lights the commuter stops for, find $P(X > 5)$.

23. Let X be a binomial random variable with mean of 2 and variance of 1.5. Find the probability that X is at least 1.

24. Let X be the number of boys in families with five children (assume that a boy or a girl is equally likely to occur and the sex of any child is independent of any brothers or sisters).

    (a) What are the possible values of the random variable X?

    (b) What is the probability that a family of five children has at least two boys?

    (c) Find the mean and standard deviation for the number of boys in families with five children.

25. Let the random variable X be the number of heads observed in the toss of three balanced coins.

    (a) Find the probability distribution of X.

    (b) Find standard deviation of X.

26. Suppose that a large lot of electrical fuses contains 5% defective. If a sample of 10 fuses is tested, find the probability of observing at least one defective.

27. In a certain section of a large city 80% of all families have at least two automobiles. In a randomly selected sample of 10 families, what is the probability that:

    (a) exactly 8 families have at least two automobiles?

    (b) more than 2 families have at least two automobiles?

28. In a tennis championship, player A competes against player B in consecutive sets, and the game continues until one player wins 3 sets. Assume that, for each set, $P(A \text{ wins}) = 0.4$, $P(B \text{ wins}) = 0.6$, and the outcomes of different sets are independent. Let X be the number of sets played.

(a) List the possible values of X.

(b) Obtain the probability distribution of X.

29. If we have to determine all the values of a binomial distribution it is sometimes helpful to calculate $f(0)$ using the formula of the binomial distribution and then calculate the other values one after one using the formula $\frac{f(x+1)}{f(x)} = \frac{(n-x)p}{(x+1)(1-p)}$. Verify the validity of this formula and then use the method explained to find all the values of the binomial distribution with $n = 4$ and $p = \frac{1}{4}$.

30. The probability that a person, living in a certain city, owns a dog is estimated to be 0.3. Find the probability that the tenth person randomly interviewed in that city is the fifth one to own a dog.

31. Find the probability that a person flipping a coin gets

    (a) the third head on the seventh flip;

    (b) the first head on the fourth flip.

32. Three people toss a fair coin and the odd man pays for coffee. If the coins all turn up the same, they are tossed again. Find the probability that fewer than 4 tosses are needed.

33. Suppose that the probability is 0.25 that a given person will believe a rumor about the private life of a certain politician. What is the probability that fifth person will be the first one to believe it?

34. Suppose that the probability is 0.70 that a child exposed to a disease will catch it. What is the probability that the third child exposed to the disease will catch it?

35. From a group of three statistician and two social scientists two persons will be selected to from a committee. Let random variable X be the number of statisticians in the committee.

    (a) Find the probability distribution of X.

    (b) Find the expected value of X.

    (c) Find the standard deviation of X.

36. An urn contains three white and two red marbles. Two marbles are drawn from this urn without replacement. Let X be the number of red marbles drawn.

    (a) Find $E(X)$.

    (b) Find $\text{Var}(X)$.

37. Suppose that an industrial organization has three vacancies of management position which are to be filled. There are 10 applicants, six of them white and four of them blacks. Let X be the number of blacks selected.

    (a) Find the probability distribution of X.

    (b) Find the expected number of blacks for these positions.

38. A warehouse contains 10 printing machines, 4 of which are defective. A company selects 5 of the machines at random. Let X be the number of non-defective machines selected. Find the probability distribution for X.

39. Out of a job population of 10 jobs with 6 jobs of class 1 and 4 of class 2, a random sample of size 5 is selected, without replacement. Let X be the number of class 1 jobs in the sample. Determine the p.d.f of X. Find μ and σ^2 of X.

40. Show that the hypergeometric probability function approaches the Binomial in limit as $N \to \infty$ and $p = \frac{k}{N}$ remains constant. That is, show that

$$\lim_{N\to\infty} \frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}} = \binom{n}{x} p^x q^{n-x}$$

for $p = \frac{k}{N}$ constant and $q = 1 - p$.

41. If a person is dealt 13 cards from an ordinary deck of 52 playing cards several times, how many hearts per hand can he expect? Between what two values would you expect the number of hearts to fall at least 75% of the time?

42. Find the probability of being dealt a bridge hand of 13 cards containing 5 spades, 2 hearts. 3 diamonds, and 3 clubs.

43. Suppose that there are on the average 63 auto accidents per week on the free ways in New York. What is the probability that

    (a) in a given day there is no auto accident?

    (b) in a given 4 days the number of auto accidents is less than 10 and greater than 5?

44. Suppose that the average number of customers who appear at a teller's window of a certain bank per minute is two. Find the probability that during a given minute

    (a) three or more customers appear

    (b) no customer appears.

45. The number of typing errors made by a particular typist has a Poisson distribution with an average of four errors per page. If more than four errors show on a given page the typist must retype the whole page. What is the probability that a certain page does not have to be retyped?

46. Person arrive in a queue at an average rate of 1 per minute. Find the probability that there will be

    (a) no arrival in the next 5 minutes.

    (b) at least one arrival in the next 5 minutes.

47. A certain automatic car wash takes exactly 5 minutes to wash a car. On the average, 10 cars per hour arrive at the car wash. Suppose that, 30 minutes before closing time, five cars are in line. If the car wash is in continuous use until closing time, what is the probability that no one will be in line at closing time?

48. Customer arrivals at a checkout counter in a department store have a Poisson distribution with a mean of 8 customers per hour. Find the probability that exactly two customers arrive between 1:00 P.M and 2:00 P.M and between 3:00 P.M and 4:00 P.M. (two separate 1-hour periods for a total of two hours)

49. In a daily production of a certain kind of rope, the number of defects per meter X is assumed to have a Poisson distribution with mean 6 defects. The profit per meter when the rope is sold is given by Y, where $Y = 50 - 2X - X^2$. Find the expected profit per meter.

50. A certain area of the eastern United States is, on average, hit by 6 hurricanes a year. Find the probability that for a given year that area will be hit by

    (a) fewer than 4 hurricanes;

    (b) anywhere from 6 to 8 hurricanes.

51. The probability that a person will die from a certain respiratory infection is 0.002. Find the probability that fewer than 5 of the next 2000 so infected will die.

52. To be a winner in the following game, you must be successful in 3 successive rounds. The game depends on the value of Y, a uniform random variable on (0,1). If $Y > 0.1$, then you are successful in round 1; if $Y > 0.2$, then you are successful in round 2; if $Y > 0.3$ then you are successful in round 3;

    (a) Find the probability that you are successful in round 1.

    (b) Find the conditional probability that you are successful in round 2, given that you were successful in round 1.

    (c) Find the conditional probability that you are successful in round 3, given that you were successful in round 1 and 2.

    (d) Find the probability that you were a winner.

53. Find the probabilities that a random variable having a normal distribution will take on a value within

    (a) one standard deviation of the mean,

    (b) two standard deviations of the mean,

    (c) three standard deviations of the mean,

    (d) four standard deviations of the mean.

54. A normal distribution has the mean $\mu = 74.4$. Find its standard deviation if 10% of the area under the curve lies to the right of 100.

55. A random variable has a normal distribution with the standard deviation $\sigma = 10$. Find its mean if the probability is 0.8264 that it will take on a value less than 77.5.

56. For a random variable having the normal distribution, the probability is 0.33 that it will take on a value less 245, and the probability is 0.48 that it will take on a value greater than 260. Find the mean and the standard deviation of this random variable.

57. Suppose that during periods of relaxation therapy the reduction of a person's oxygen consumption may be looked upon as a random variable having a normal distribution with $\mu = 38.6$ cc per minute and $\sigma = 4.3$ cc per minute. Find the probabilities that during a period of relaxation therapy a person's oxygen consumption will be reduced by

    (a) at least 40.0 cc per minute,

    (b) anywhere from 35.0 to 45.0 cc per minute.

58. In a test given to a large group of people, the scores were normally distributed with mean 70 and standard deviation 10.

    (a) What percentage of people can get a score less than 50?

    (b) What is the least whole-number score that a person could get and yet score in about the top 15%?

59. The weekly amount spent for maintenance and repairs in a certain company was observed, over a long period of time, to be approximately normally distributed with a mean of \$400 and a standard deviation of \$20.

    (a) If \$450 is budgeted for the next week, what is the probability that the actual costs will exceed the budgeted amount?

    (b) How much should be budgeted for weekly repairs and maintenance in order that the budgeted amount should be exceeded with a probability of only 0.1?

60. The grade point averages (GPA) of a large population of college students are approximately normally distributed with a mean of 2.4 and a standard deviation of 0.8.

    (a) What percentage of students will possess a grade point average in excess of 3.0?

    (b) What percentage of students will possess a GPA less than 2.0?

    (c) What must be the minimum GPA to be in the top 10%?

61. A city water department finds household water use to be normally distributed with a mean of 27 gallons and a standard deviation of 3 gallons per day.

    (a) Find the probability that a randomly chosen household uses more than 25 gallons per day, uses between 20 and 30 gallons per day.

    (b) If the mayor wants to give a tax rebate to the 17-percent lowest water users, what should the gallons-per-day cutoff be?

62. Scores on an examination are assumed to be normally distributed with a mean of 78 and a variance of 36.

    (a) What is the probability that a person taking the examination scores higher than 72?

    (b) Suppose that students scoring in the top 10% of this distribution are to receive a letter grade AA. What is the minimum score a student must achieve to earn an AA?

(c) What must be the cutoff point for passing the examination if the examiner wants only the top 28.1% of all scores to be passing?

(d) Find approximately what proportion of students have scores 5 or more points above the score that cuts of the lowest 25%.

63. An aptitude test administered to aircraft pilot trainees requires a series of operations to be performed in quick succession. Suppose that the time needed to complete the test is normally distributed with mean 90 minutes and standard deviation 20 minutes.

   (a) To pass the test, a candidate must complete it within 80 minutes. What percentage of the candidates will pass the test?

   (b) If the top 5% of the candidates are to be given a certificate of commendation, how fast must a candidate complete the test to be eligible for a certificate?

64. At a local discount store, service times at the checkout counter are observed to be normally distributed with mean 3.5 minutes and variance of 1.44.

   (a) Find the probability that a customer takes more than 5 minutes to check out.

   (b) A customer has been checking out for 3 minutes. What is the probability that it will take at least 5 minutes for the entire process?

   (c) What is the probability that next 6 customers check out in a total of 20 minutes or less?

65. The number of calories in a salad on the lunch is normally distributed with mean 200 and a standard deviation 5. Find the probability that the salad you select will contain

   (a) more than 208 calories.

   (b) between 190 and 200 calories.

66. Assume the scores made on a real estate broker examination are normally distributed with mean 200 and standard deviation 20. Any person making a score in excess of 230 points on the examination becomes a certified real estate broker. Assume 10 persons take the exam simultaneously and the scores they make, $X_1, X_2, \ldots, X_{10}$ are a random sample from this population of possible scores.

   (a) What is the probability that any one of these persons will be certified?

   (b) What is the probability that exactly one of the 10 will be certified?

   (c) What is the probability that at least one of the 10 will be certified?

67. A soft drink machine can be regulated so that it discharges an average of μ ounces per cup. If the ounces of fill are normally distributed with standard deviation equal to 0.3 ounce, give the value of μ so that 8-ounce cups will overflow only 1% of the time.

68. The width of bolts of fabric is normally distributed with a mean of 950 millimeters and a standard deviation of 10 millimeters.

   (a) What is the probability that a randomly chosen bolt has a width between 947 and 658 millimeters?

(b) What is the approximate value c such that a randomly selected bolt has a width less than c with probability 0.8531?

69. The pulse rate per minute of the adult male population between 18 and 25 years of age in the United States is known to have a normal distribution with a mean of 72 beats per minute and a standard deviation of 9.7. If the requirements for military service state that anyone with a pulse rate over 100 is medically unsuitable for the service, what proportion of the males between 18 and 25 years age would be declared unfit because their pulse rates are too high?

70. A soda filling operation is designated to fill cans with average 12 ounces of soda, with a standard deviation of 0.4 ounce. Assume that the amount of fill is normally distributed. What is the probability that a randomly selected can will contain less than 11 ounces.

71. In a large corporation number of sales contacts per week are normally distributed with standard deviation of 2 and a mean of 17 contacts. Find the probability that a randomly selected salesperson makes

    (a) more than 15 sales per week?

    (b) less than 10 sales per week?

72. The time it takes a symphony orchestra to play Beethoven's Ninth Symphony has a normal distribution with mean of 64.3 minutes and a standard deviation of 1.15 minutes. The next time it is played, what is the probability that it will take between 62.5 and 67.7 minutes?

73. An efficiency expert knows that the time required by electricians to wire a certain type of house is normally distributed with mean of 14 hours and standard deviation of 1.3 hours. Evaluate these claims by two electricians:

    (a) "I can wire a house like this in 11 hours."

    (b) "It took me 17 hours of hard work to wire this house."

74. A study of 50,000 coal workers shows that the daily output per worker equals 17.3 tons on the average. Some 89.25 percent produces 18.1 tons or fewer. What percentage produces more than 16 tons? (Hint: assume the daily output of a worker is normally distributed).

75. The height, X, that a college jumper will clear each time she jumps is normal with mean 180 cm and standard deviation 4 cm.

    (a) What is the probability that the jumper will clear 190 cm on a single jump?

    (b) Assuming the jumps are independent, what is the probability that 190 cm will be cleared on exactly 3 of the next 4 jumps?

76. Scores on a hearing test are normally distributed with a mean of 600 and a standard deviation of 100.

    (a) If one subject is randomly selected, find the probability that the score is more than 450.

    (b) If a job requires a score in the top 80%, find the lowest acceptable score.